Performance Analysis of the Weighted Window CFAR Algorithms

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***Abstract*—With the deterioration of radar operation environment and the enhancement of menace to radar, the task of radar targets detection becomes more complicated. Such as the detection of airplane, ship or cruise missile in over the horizon radar (OTHR), and the detection of the moving targets in synthetic aperture radar (SAR). Therefore, it's necessary to make further study on CFAR algorithms. The performance of conventional cell averaging (CA) algorithm is the best in homogeneous background since it uses the maximum likelihood estimate of the noise power to set the adaptive threshold. But if the interfering target is present in the reference window with a target return in the test cell, sever masking of targets appears due to increased threshold. In order to overcome this problem, the ordered statistic (OS) and the trimmed mean (TM) algorithms using trimmed technique are proposed. If the reference sample number is not too big, the CFAR loss of OS and TM increase greatly. This case can usually be encountered in complicated environment and lower SNR situation. In this paper, weighted window techniques such as rectangle, steps and trapezium windows are discussed. The analysis results show that weighted window technique can improve greatly in homogeneous background and obtains an immune ability to interfering targets to some extent.**

I. INTRODUCTION

With the deterioration of radar operation environment and the enhancement of menace to radar, the task of radar targets detection becomes more complicated. Such as the detection of airplane, ship or cruise missile in over the horizon radar (OTHR), and the detection of the moving targets in synthetic aperture radar (SAR). Therefore, it's necessary to make further study on CFAR algorithms. The performance of conventional cell averaging (CA) algorithm is the best in homogeneous background since it uses the maximum likelihood estimate of the noise power to set the adaptive threshold. But if the interfering target is present in the reference window with a target return in the test cell, sever masking of targets appears due to increased threshold. In order to overcome this problem, the ordered statistic[1] (OS) and the trimmed mean[2] (TM) algorithms using trimmed technique are proposed. In practical applications, the information about the number of interfering targets is not known in advance, the biggest samples of reference window are always trimmed with OS or TM method not only in multiple targets situation but also in homogeneous background, this will results additional CFAR loss. Especially in the case of the short reference window, this CFAR loss will becomes unacceptable, and this can usually be encountered in complicated environment and lower SNR situation. In this paper, the weighted window techniques such as rectangle, steps and trapezium windows are discussed. The analysis results show that the weighted window technique can improve greatly in homogeneous background and obtains an immune ability to interfering targets.

II. GENERAL DETECTION PRINCIPLE

In this paper we assume that the background noise envelope is Rayleigh distributed, the test cell and the reference cell variables are independent, we only consider the single pulse square-law detection. In homogeneous background, the reference samples are IID with common probability density function (pdf) and cumulative distribution function (cdf)

$$f(x)=\frac{1}{\mu}exp(-\frac{x}{\mu}),$$
$$F(x)=1-exp(-\frac{x}{\mu}) \qquad (x>0) \tag{1}$$

μ represents noise power.

The detector in this paper forms a finite reference window of size M surrounding the cell under test and uses the reference samples that fall within the window to estimate the total noise power. The returns in the reference cells X_M, X_{M-1}, …, X_3, X_2, X_1 are sorted according to their amplitudes, the ordered samples are obtained as

$$x_{(M)} \geq x_{(M-1)} \geq \cdots \geq x_{(3)} \geq x_{(2)} \geq x_{(1)} \tag{2}$$

The weighted sum of ordered samples is used as a noise power estimation

$$X=\sum_{k=1}^{M} h_k x_{(k)} \tag{3}$$

There, the mean of the kth ordered sample is[3]

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$$\overline{x(k)} = \mu \sum_{i=1}^{K} \frac{1}{N+1-i} \tag{4}$$

and the estimation variance of μ using $x_{(k)}$ is obtained as

$$\sigma^2 = D\hat{\mu} = E(\hat{\mu} - \mu)^2 = \mu^2 \frac{\sum_{i=1}^{k} \frac{1}{(N+1-i)^2}}{\left(\sum_{j=1}^{k} \frac{1}{(N+1-j)}\right)^2} \tag{5}$$

When the number of the reference sample is 24, Fig.1 depicts the mean curve of $X_{(k)}$ versus k, it's shown that the mean of higher ordered sample is bigger and that of the lower ordered sample is smaller. Fig.2 depicts the estimation variance curve of μ using $X_{(k)}$ versus k, it's shown that the variance of higher ordered sample is smaller than that of lower ordered sample, but the variance of the biggest ordered sample is not the smallest.

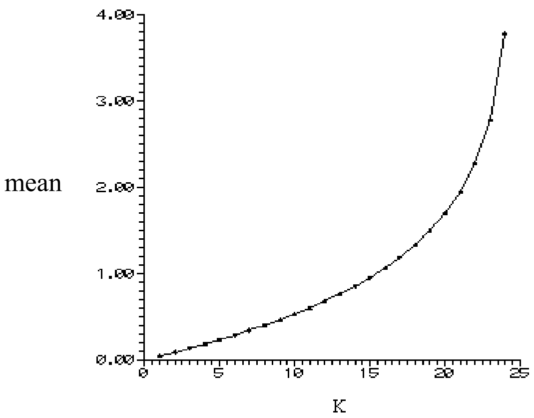

Fig.1 The curve of the mean value of $X_{(k)}$ versus k.

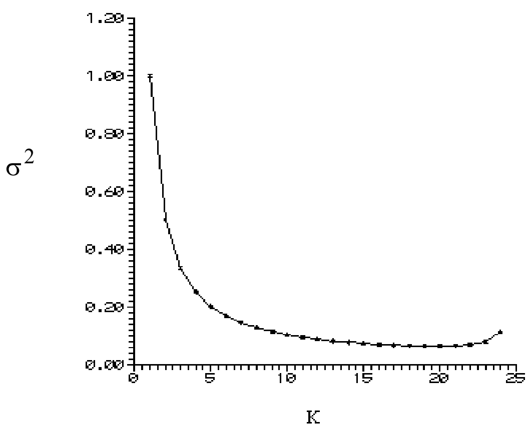

Fig.2 The estimation variance curve using $X_{(k)}$ versus k.

Finally, the moment generation function (mgf) of X in (3) can be obtained as

$$\Phi_X(u) = \prod_{i=1}^{M} \frac{c_i}{\mu u + c_i} \qquad \left(c_i = \frac{M+1-i}{\sum_{k=i}^{M} h_k}\right) \tag{6}$$

The noise power estimation X is multiplied by a threshold constant and then compared with the test cell return. If the test cell return exceeds this threshold, a detection is declared. The noise power estimation X which sets threshold is a random variable, thus the detector performance is determined by average detection and false alarm probability, average detection probability and false alarm probability is respectively

$$P_{fa} = \Phi_X(u)\Big|_{u=\frac{T}{\mu}} \qquad P_d = \Phi_X(u)\Big|_{u=\frac{T}{b\mu}} \tag{7}$$

$\Phi_X(u)$ is mgf of X, b=1+λ, λ is the per pulse average SNR.

The analytic expressions of average detection threshold (ADT) is obtained as

$$ADT = -\frac{T}{\mu}\frac{d\Phi_X(u)}{du} = T\sum_{i=1}^{M}\frac{1}{c_i} \tag{8}$$

III. PERFORMANCE ANALYSIS OF THE WEIGHTED WINDOW ALGORITHMS

In order to enhance the ability of the detector to combat the interfering targets, OS, CM, TM, the best linear unbiased[4] (BLU) and the quasi best weighted[5] (QBW) algorithms use trimming method to trim the bigger or smaller ordered samples. But if the length of the reference window is not too big, this trimming method will results the more additional CFAR loss with respect to CA that averages all reference samples. Furthermore, this short reference window which is confined by the practical situation is usually encountered in complicated radar detection environment and low SNR case. In order to improve the detection performance and enable the detector to have an immune ability to interfering targets, the weighted window techniques such as rectangle, steps and trapezium windows are presented in this paper, and we give some analytic results with comparison to other schemes. Some CFAR methods such as CA, CM, OS, TM, BLU and QBW can be obtained by some special cases of (3). If the weighted coefficients h_i selects as $h_i = 1/M (i = 1, \cdots, M)$, that is CA method. If h_i selects as $h_i = 1/(M - M_2)(i = 1, \cdots, M - M_2)$, others are zero, that is CM method. If h_i selects as $h_i = 1/(M - M_2 - M_1)(i = M_1 + 1, \cdots, M - M_2)$, others are zero, that is TM method. If h_i select as $h_i = 1 (i = k)$, others are zero, that is OS method. If h_i selects as

$$h_i = \frac{1}{\Delta}\begin{cases} 1 & i = M_1 + 2, \cdots, M - M_2 - 1 \\ -M + M_1 + 1 + \dfrac{\sum_{j=0}^{M_1} (M-j)^{-1}}{\sum_{j=0}^{M_1} (M-j)^{-2}} & i = M_1 + 1 \\ M_2 + 1 & i = M - M_2 \end{cases}$$

$$\left(\Delta = M - M_1 - M_2 - 1 + \frac{\left(\sum_{j=0}^{M_1}(M-j)^{-1}\right)^2}{\sum_{j=1}^{M_1}(M-j)^{-2}}\right) \tag{9}$$

the mathematical model of BLU can be obtained.

If h_i of expression (3) selects as

$$h_i = \frac{1}{M - M_2 - M_1}\begin{cases} 1 & i = M_1 + 1, \cdots, M - M_2 - 1 \\ M_2 + 1 & i = M - M_2 \\ 0 & else \end{cases} \tag{10}$$

the method of QBW can be obtained.

Now we consider the weighted windows technique, in the first case, the rectangle window

$$h_i = \frac{1}{M - M_2 - M_1}\begin{cases} 1 & M_1 + 1 \le i \le M - M_2 \\ 0 & 1 \le i \le M_1 \, or \quad M - M_2 + 1 \le i \le M \end{cases} \tag{11}$$

In fact, CA, OS, CM and TM are the special cases of rectangle window.

The second case, steps window

$$h_i = \frac{1}{M_1\alpha + M_2\beta + M - M_1 - M_2}\begin{cases} \alpha & 1 \le i \le M_1 \\ 1 & M_1 + 1 \le i \le M - M_2 \\ \beta & M - M_2 < i \le M \end{cases} \tag{12}$$

$(0 \le$

Because the interfering targets usually occupy the higher order statistics and the null sample will occupy the lower order statistics, the weighted coefficient α and β of higher and lower order satistics take smaller, rather than take zero in the case of TM.

The third case also from the view point of the smaller coefficient of higher and lower order statistics, the coefficients of trapezium window is

$$h_i = \frac{w_i}{\sum_{j=1}^{M} w_j} \tag{13}$$

where

$$w_j = \begin{cases} \frac{1-\alpha}{M_1}(j-1) + \alpha & 1 \le j \le M_1 \\ 1 & M_1 + 1 \le j \le M - M_2 \\ -\frac{1-\beta}{M_2}(j - M) + \beta & M - M_2 < j \le M \end{cases}$$

$$(0 \le \alpha \le 1, \quad 0 \le \beta \le 1) \tag{14}$$

TABLE I. The performance comparison of detection schemes in homogenious background.

Schemes	12 (the sample number)		16(the sample number)	
	CFAR loss	ADT	CFAR loss	ADT
CA	2.7137	25.9473	1.9957	21.9420
OS	(k=9)3.9119	34.4710	(k=13)2.7481	26.2326
CM(M2=3)	3.8560	33.9621	2.5973	25.2668
TM(M2=3, M1=3)	3.8149	33.6232	2.5816	25.1721
BLU(M2=3, M1=3)	3.7185	32.8405	2.4925	24.6404
QBW(M2=3, M1=3)	3.7261	32.9022	2.4954	24.6575
Rectangle window (M2=3, M1=3)	3.8149	33.6232	2.5816	25.1721
Steps window (M2=3, M1=3, α=1/8,β=1/8)	3.1360	28.7023	2.2899	23.5218
Trapezium window (M2=3, M1=3α=1/8,β=1/8)	2.9569	27.4985	2.1599	22.8105

TABLE II. The ADT and CFAR loss of steps window versus α and β.（$M = 16, M_2 = 3, M_1 = 3$）

	α=1/10,β=1/10	α=1/8,β=1/8	α=1/4,β=1/4	α=1/2,β=1/2
ADT	23.7462	23.5218	22.7805	22.1716
CFAR loss	2.3305	2.2899	2.1536	2.0393

TABLE III. The ADT and CFAR loss of trapezium window versus α and β.（$M = 16, M_2 = 3, M_1 = 3$）

	α=1/10,β=1/10	α=1/8,β=1/8	α=1/4,β=1/4	α=1/2,β=1/2
ADT	22.9050	22.8105	22.4592	22.1058
CFAR loss	2.1776	2.1599	2.0938	2.0269

The performance analysis of the weighted window techniques and other schemes are carried out by their CFAR loss and ADT values. Table I shows the performance of the weighted window technique and other schemes in homogeneous background. It's can be seen that an evident improvement is obtained through weighted window techniques in homogeneous background, and the influence of interfering targets can be counteracted due to the smaller coefficients of higher ordered sample. For example, the number of reference samples is 12, the CFAR loss of CA is 2.7137, the CFAR loss of OS at k=9 is 3.9119, the CFAR loss of steps window and trapezium window at $\alpha=\beta=1/8$ is 3.1360 and 2.9569 respectively. Table II and table III give the CFAR loss and ADT values of steps window and trapezium window versus α and β respectively. It can be shown that with the increase of α and β, the CFAR loss and ADT decrease, if $\alpha=\beta=1$, the scheme becomes CA. Because of the simplicity of CA in their implementation, it is commonly used in radar systems. But in the cases of multiple targets, the "masking effect" of CA will appears. OS and TM methods can resolve the multiple targets well, they also bring some additional CFAR loss, especially in the case of the short reference window. The weighted window technique discussed here can be considered as a compromise between CA and TM.

IV. CONCLUSION

In order to improve the detection performance of OS and TM in the case of the short reference window, the weighted window techniques are proposed in this paper. The weighted window technique can be considered as a compromise between CA and TM. In the cases of the low SNR situation and the short reference window, applying weighted window techniques are of practical interest. It's shown that an evident improvement is obtained through the weighted window techniques in homogeneous background, and the influence of interfering targets can be counteracted due to the smaller coefficients of higher ordered samples. With the increase of α and β, the CFAR loss and ADT decrease, if $\alpha=\beta=1$, the scheme becomes CA. The smaller α and β are, the stronger ability to combat the interfering targets. It's shown from Fig.2 that the variance of higher ordered sample is smaller than that of lower ordered sample, but the variance of the biggest ordered sample is not the smallest. Therefore in practical applications, the upper-medium parts of weighted coefficients should be bigger, whereas the two ends of ordered samples should be smaller.

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